International Journal of Scientific Research and Modern Education (IJSRME)
Impact Factor: 7.137, ISSN (Online): 2455 – 5630
(www.rdmodernresearch.com) Volume 4, Issue 2, 2019

STOCHASTIC MODEL ON FINDING STATIONARY INCREMENTS OF WATER LEVEL IN METTUR DAM DURING JUNE 2005-MAY 2006 USING GAUSSIAN PROCESSES

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Cite This Article: P. Senthil Kumar & A. Arul Selvi, "Stochastic Model on Finding Stationary Increments of Water Level in Mettur Dam During June 2005-May 2006 Using Gaussian Processes", International Journal of Scientific Research and Modern Education, Volume 4, Issue 2, Page Number 7-14, 2019.

Abstract:

In India water resources are increasingly becoming scarce. Since rainfall occurs only for three months in a few spells, storage by dams is imperative to utilize waters defined in [1]. The Mettur dam is one of the largest dams in India and the largest in Tamil Nadu. This study examines the everyday inflow and outflow water level in Mettur dam from June 2005 to May 2006 and exhibits that whether the water is used by the farmers for their Irrigation or not. In this paper, we expose the functional convergence theorems for quadratic variations of second order of Gaussian progression with singularity function and the stationary increments of Gaussian progression and compare with the water level in Mettur dam.

Key Words: Water Level, Inflow, Outflow, Almost Sure Convergence, Generalize Quadratic Variations & Gaussian Progression

1. Introduction:

Mettur dam was developed in a gorge, where the river Cauvery enters the field. It is constructed mostly for power production, irrigation water supply or flood preclusion. It offers water system offices to parts of Salem, Namakkal, Erode, Trichirappalli, Karur, and Thanjavur districts for 2,71,000 sections of land. Thousands of acres of land in the region of Salem are irrigated with the help of this dam. Irrigation water can be stored in reservoirs during the wet season, and in the arid seasons it can be released from the dam. The Mettur dam has received public attention, since the latter half of the 20th century, and especially in the mid 1990's due to the Cauvery river water dispute between the states of Tamil Nadu and Karnataka.

In [8], we have already studied the convergence of the sequence of second order quadratic variations. An almost sure asymptotic expansion and a central limit theorem for the sequence of second order quadratic variations had proved in [2]. The localized quadratic variations have been introduced in [5] to construct the estimators of the Hurst function of multi fractional Gaussian progression.

Consider a Gaussian progression $X=\{X_t; t\in[0,1]\}$ and denote its mean function by M and its covariance function by C,

$$M(t)=E(X)_t, \quad \forall t\in[0,1]$$

$$C(s,t)=E((X_t-M_t)(X_s-M_s)) \qquad \forall s,t\in[0,1]$$

Denote the second order quadratic variations

$P_n(X)=\{P_n(X)_t; t\in[0,1]\}$, and it is defined by

$$P_n(X)_t=\sum_{r=1}^{[(n-1)t]}\left[X_{\frac{r+1}{n}}+X_{\frac{r-1}{n}}-2X_{\frac{r}{n}}\right]^2$$

The trajectory of the process be in the space of real functions defined of $[0,1]$ that is $([0,1])$ $\mathcal{D}$and defined in [3] also classify the linear interpolations

$$\gamma_n(X)_t=P_n(X)_t+[(n-1)t-((n-1)t)](\Delta X^n_{((n-1)t)+1})^2,$$

Where $1\le r\le n-1$, at the point $\frac{k}{n}$, the second order increment of X is denoted by ΔX^n_r,

$$\Delta X_r^n = X_{\frac{r+1}{n}} + X_{\frac{r-1}{n}} - 2X_{\frac{r}{n}}$$

Then the trajectories of this progression belong to the space of continuous real functions defined of $[0,1]$ that is

$$\zeta([0,1])$$

Signify the second order localized quadratic variations by

$$\left(P_n^{loc}(X)_t\right) = \sum_{r \in V_n(X)_t} \left[X_{\frac{r+1}{n}} + X_{\frac{r-1}{n}} - 2X_{\frac{r}{n}} \right]^2, \ \forall t \in [0,1]$$

2. Functional Convergence of $P_n(X)$ And $\gamma_n(X)$

Definition 1:

A Borelian function $\varphi :]0,a[\to R(a>0)$ is regularly varying of index $\lambda \in R if \varphi(h) = h^{\lambda}U(h)$, where U is slowly varying function $\lim_{x \to 0} \frac{U(\beta x)}{U(x)} = 1$, for all $\beta > 0$. The symbol of convergence in law [7] is denoted by $\overset{U}{\to}$.

Theorem 1:

Let a Centred Gaussian progression X, satisfy the conditions

a) C is continuous on $[0,1]^2$.

b) Let $G = (0 \le t \le s \le 1)$ and the derivative $\frac{\partial^4 c}{\partial s^2 \partial t^2}$ exists on $]0,1]^2$ and there is a continuous function $F : G \to R$ a real $\eta \in]0,2[$ positive slowly varying function $U : [0,1] \to R$ such that

$$\frac{(s-t)^{2+\eta}}{U(s,t)} \frac{\partial^4 c}{\partial s^2 \partial t^2}(s,t) = F(s,t) \quad , \qquad \text{for all } s,t \in G.$$

Where the interior of G is denoted by $\dot{G}$, that is $\dot{G} = (0 < s < t < 1)$.

c) Assume there exist $r+1$ functions $u_0, u_1, \ldots\ldots u_r$ from $]0,1[to R \quad 0 < v_1 < v_2 < \ldots\ldots < v_r$ of r real numbers and there exists a function $\varphi :]0,1[\to]0,\infty[$ Such that

i) If $r \ge 1$ then $\forall 0 \le i \le r-1$, u_i is Lipschitz on $]0,1[$,

ii) The u_r is equal to $\frac{1}{2} + \in_r$ holds on $]0,1[$ with $0 < \in_r \le \frac{1}{2}$,

iii) There exists $t \in]0,1[$ such that $u_0(t) \ne 0$

iv) $$\lim_{h \to 0} \frac{1}{\sqrt{h}} \left(\sup_{h<t<1-h} \left| \frac{\left(\delta_1^h \circ \delta_2^h c\right)}{h^{2-\eta}U(h)} - u_0(t) - \sum_{i=1}^{r} u_{i(t)} \varphi(h)^{v_i} \right| \right) = 0$$

Where $\sum_{i=1}^{r} u_{i(t)} \varphi(h)^{v_i} = 0$ if $r = 0$ and if $r \ne 0$, $\lim_{h \to 0} \varphi(h) = 0$

v) There exist a bounded function $\tilde{u} :]0,1[\to R$ such that

$$\lim_{h \to 0} \left(\sup_{h<t<1-2h} \left| \frac{\left(\delta_1^h \circ \delta_2^h c\right)(t+h,t)}{h^{2-\eta}U(h)} - \tilde{u}(t) \right| \right) = 0$$

Then almost surely and uniformly in $t \in [0,1]$

$$\lim_{n \to \infty} \frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)} P_n(X)_t = \int_0^t u_0(x)\,dx \tag{1}$$

and the process

$$\left\{\sqrt{n}\left(\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}P_n(X)_t-\int_0^t u_0(x)dx-\sum_{i=1}^{r}\int_0^t u_{i(x)}dx\varphi(1/n)^{v_q}\right)\right\} \tag{2}$$

Converges in law when n tends to infinity and towards a Gaussian process

$$Z_t=\int_0^t\left(2u_0(x)^2+4\tilde{u}(x)^2+4\left(\|\rho_\eta\|\right)^2F(x,x)^2\right)^{1/2}dN_x \tag{3}$$

Where N is a standard Brownian motion

Proof:

Choose the assumption $v_0=0$ and set the functions for all $t\in[0,1]$

$$d_n(t)=\sum_{i=1}^{r}\int_0^t u_{i(x)}dx\varphi(1/n)^{v_i}$$

$$G_n(t)=\sqrt{n}\,\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}P_n(X)_t$$

$$\tilde{G}_n(t)=G_n(t)-E(G_n(t))$$

Step 1:

In [2], the equation (1) is proved for the case $t=1$ and the related arguments hold for any $t\in[0,1]$ and uniformity of (1) is a consequence of Helly theorem done in [7].

Step 2:

Suppose that, $0\le s\le t\le 1$.

Assume $\tau_{s,t}=\int_s^t\left(2u_0(x)^2+4\tilde{u}(x)^2+4\left(\|\rho_\eta\|\right)^2F(x,x)^2\right)dx$

By the significance of the proof of theorem 5 in [2],

We attain it, $\tilde{G}_n(t)-\tilde{G}_n(s)\xrightarrow{U}(0,\tau_{s,t})$, when $n\to\infty$.

That is, $$\lim_{n\to\infty}\operatorname{var}\left(\tilde{G}_n(t)-\tilde{G}_n(s)\right)=\tau_{s,t} \tag{4}$$

Consider the variable of one dimension,

$$S_n(\alpha_1,\alpha_2)=\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}\alpha_1P_n(X)_s+\alpha_2P_n(X)_t\ ,$$

Here α_1,α_2 are non negative real numbers.

From the asymptotic property, we deduce that

$$\begin{aligned}\operatorname{var}(S_n(\alpha_1,\alpha_2))&=\frac{1}{n}\alpha_1^{\,2}\operatorname{var}\left(\tilde{G}_n(s)\right)+\alpha_2^{\,2}\operatorname{var}\left(\tilde{G}_n(t)\right)+2\alpha_1\alpha_2\operatorname{var}\left(\tilde{G}_n(s)\right)\\&+2\alpha_1\alpha_2\operatorname{cov}\left(\tilde{G}_n(s),\tilde{G}_n(t)-\tilde{G}_n(s)\right)\end{aligned} \tag{5}$$

and with the formula (4) it yields

$$\lim_{n\to\infty}\operatorname{cov}\left(\tilde{G}_n(s),\tilde{G}_n(t)-\tilde{G}_n(s)\right)=0 \tag{6}$$

Accordingly, formula (5) and (6) yields

$$\begin{aligned}\lim_{n\to\infty}n\operatorname{var}(S_n(\alpha_1,\alpha_2))&=\alpha_1^{\,2}\operatorname{var}(Z_s)+\alpha_2^{\,2}\operatorname{var}(Z_t)+2\alpha_1\alpha_2\operatorname{var}(Z_s)\\&=\operatorname{var}(\alpha_1Z_s+\alpha_2Z_t)\end{aligned} \tag{7}$$

Applies the Lindeberg central limit theorem and consider $S_n(\alpha_1,\alpha_2)$ as the Euclidean norm of the Guassian vector $(Y_i : 1 \le i \le [t(n-1)])$, with

$$Y_i = \sqrt{\alpha_1+\alpha_2}\sqrt{\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}}\Delta X_i^n, \qquad 1 \le i \le [s(n-1)]$$

$$Y_i = \sqrt{\alpha_2}\sqrt{\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}}\Delta X_i^n, \qquad [s(n-1)] \le i \le [t(n-1)]$$

Hence by the classical Cochran theorem, we can find c_n positive real numbers $(\sigma_{1.n},\sigma_{2,n}\ldots\ldots\sigma_{cn.n})$ and one c_n dimensional Gaussian vector ε_n, and

$$S_n(\alpha_1,\alpha_2)=\sum_{i=1}^{c_n}\sigma_{i,n}\left(\varepsilon_n^i\right)^2$$

Since in the proof of (30) in [4], we obtain that, $\sigma_n^* = \max\limits_{1\le i\le c_n}\sigma_{i,n}$

Accordingly with (7) implies that,

$$\lim_{n\to\infty}\frac{\sigma_n^*}{\sqrt{\operatorname{var}(S_n(\alpha_1,\alpha_2))}}=0$$

And also apply Lindeberg central limit theorem which implies that,

$$\sqrt{n}\left(S_n(\alpha_1,\alpha_2)-E(S_n(\alpha_1,\alpha_2))\right)\xrightarrow{U}\alpha_1 Z_s+\alpha_2 Z_t,$$

Therefore $\tilde{G}_n(s)+\alpha_2\tilde{G}_n(t)\xrightarrow{U}\alpha_1 Z_s+\alpha_2 Z_t$, when $n\to\infty$.

From this we proved the convergence of the finite dimension margin of the process $\tilde{G}_n$ towards Z.

From [6] we obtain that, for all $0 \le t_1 \le t_2 \le 1$,

$$E\left(\left|\tilde{G}_n(t)-\tilde{G}_n(t_1)\right|^2\left|\tilde{G}_n(t_2)-\tilde{G}_n(t)\right|^2\right)\le K\left|t_2-t_1\right|^2 \tag{8}$$

For large n, where K is a positive constant and we have either $\tilde{G}_n(t)=\tilde{G}_n(t_1)$

If $t_2-t_1\le\frac{1}{n}$ for large n and (8) is true for any K

If $t_2-t_1\ge\frac{1}{n}$ then we apply Cauchy's inequality, for all $0\le x\le y\le 1, y-x\ge\frac{1}{n}$

We obtain from that

$$E\left(\left|\tilde{G}_n(y)-\tilde{G}_n(x)\right|^4\right)\le K|y-x|^2 \tag{9}$$

This implies that

$$G_n(y)-G_n(x)=\sqrt{n}\,\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}\sum_{r=[(n-1)x+1]}^{[(n-1)y]}\Delta X_r^{n\,2}$$

From the Cochran theorem, we obtain $c_n(x,y)$ positive real numbers $(\sigma_{1.n}(x,y),\sigma_{2,n}.(x,y)\ldots..\sigma_{cn.n}(x,y))$ and one $c_n(x,y)$ dimensional Gaussian vector $\varepsilon_n(x,y)$, such that,

This implies that $\tilde{G}_n(y)-\tilde{G}_n(x)=\sqrt{n}\sum\limits_{j=1}^{c_n(x,y)}\sigma_{j,n}(x,y)\left(\varepsilon_n{}^j(x,y)-1\right)^2$

By lemma 4.4 in [9], $E\left(\left|\tilde{G}_n(y)-\tilde{G}_n(x)\right|^4\right)\le K_n^2\left(\sum\limits_{j=1}^{c_n(x,y)}\sigma_{j,n}(x,y)^2\right)^2$

$$E\left(\left|\tilde{G}_n(y)-\tilde{G}_n(x)\right|^4\right) \le K_n^2 c_n(x,y)^2 \sigma_n^{*}(x,y)^4$$

Where $$\sigma_n^{*}(x,y) = \max_{1\le j\le c_n} \sigma_{j,n}(x,y)$$

By the preposition 30 in [4],

We obtain that, $\sigma_n^{*}(x,y) \le K_n^{-1}$

Where K is independent of (x,y) , $c_n(x,y)$ is less than or equal to the dimension of the vector ΔX_r^n .It follows that

$$c_n(x,y) \le (y[n-1]-x[n-1])$$

Consequently,

$$E\left(\left|\tilde{G}_n(y)-\tilde{G}_n(x)\right|^4\right) \le \frac{K}{n^2}(y[n-1]-x[n-1])^2$$

$$E\left(\left|\tilde{G}_n(y)-\tilde{G}_n(x)\right|^4\right) \le \frac{K}{n^2}([n-1][y-x]+1)^2$$

In this case, we take $y - x \ge \frac{1}{n}$.

It follows that (9) is satisfied. This proves the tightness of the family $\left(\tilde{G}_n\right)$ in the space $D([0,1])$ .Next to prove the theorem, we use the decomposition

$$G_n(t)-\sqrt{n}d_n(t)=\tilde{G}_n(t)+E(G_n(t))-\sqrt{n}d_n(t)$$

By the theorems 3 and 5 in [2] for the case $t = 1$, we obtain that

$$\lim_{n\to\infty}\sup_{0\le t\le 1}\left|E\left(G_n(t)-\sqrt{n}d_n(t)\right)\right| = 0 \tag{10}$$

Combining the convergence of the process $\tilde{G}_n$ towards Z and the tightness of the family $\left(\tilde{G}_n\right)$ with Prokhorov theorem, (2) is satisfied.

Corollary 1:

Let X be a Gaussian process .Assume i) the paths of X are $(1-\eta/2-y)$ holds on $]0,1[$ for all $0<y<1-\eta/2$.

ii) $\lim_{n\to\infty}\sqrt{n}\,U\left(\frac{1}{n}\right)=\infty$ .Then the process

$$\left[\sqrt{n}\,\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}P_n(X)_t - \int_0^t u_0(x)dx - \sum_{i=1}^{r}\int_0^t u_{i(x)}dx\,\varphi(1/n)^{v_i}\right]$$

It converges as $n \to \infty$ on $\zeta([0,1])$ towards Z defined by formula (3).

Proof:

Using the notations of previous theorem we obtain that

$$\sqrt{n}\left(\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}\gamma_n(X)_t - d_n(t)\right) = \sqrt{n}\left(\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}P_n(X)_t - \frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}E(P_n(X)_t)\right) + EG_n(t)-\sqrt{n}d_n(t) \tag{11}$$

$$[(n-1)t-[(n-1)t]]\sqrt{n}\,\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}\Delta X^{n}_{((n-1)t)+1}{}^{2}$$

The first term of the right- hand side of (11) converges to Z when $n \to \infty$ in the space $D([0,1])$. Limit (10) implies that the second term of the right- hand side of (11) converges to 0 when uniformly in $t \in [0,1]$ .consequently, this process implies that

$$\sqrt{n}\left(\frac{n^{1-\eta}}{U\left(\frac{1}{n}\right)}\gamma_n(X)_t - d_n(t)\right),\ t \in [0,1]$$

converges towards Z in the space, $D([0,1])$ when $n \to \infty$

3. Gaussian Progression with Stationary Increments:

Assume that the Gaussian progression X has stationary increments, the functions $u_{0,}\tilde{u}$ and $t \to \zeta(t,t)$ are constant. Therefore the process Z defined in (3) is equal to the standard Brownian motion. If X is the standard fractional Brownian motion with Hurst index $\mathrm{H} \in]0,1[$ That is,

$$C(s,t) = \frac{\left(s^{2\mathrm{H}} + t^{2\mathrm{H}} - |s-t|^{2\mathrm{H}}\right)}{2}\quad,\quad \forall s,t \in R$$

$$Z_t = \tau_{FBM\mathrm{H}} N_t\ ,\quad \forall t \in [0,1]$$

Where $\tau^2{}_{FBM\mathrm{H}} = 2\left(4-2^{2\mathrm{H}}\right)^2 + \left(2^{2\mathrm{H}+2} - 7 - 3^{2\mathrm{H}}\right)^2 + 2\mathrm{H}^2(2\mathrm{H}-1)^2(2\mathrm{H}-2)^2(2\mathrm{H}-3)^2\left|\rho_{2-2\mathrm{H}}\right|^2$

It has been defined in [2].

4. Example:

The Mettur dam receives inflow from its own catchment area situated in Karnataka and it provides irrigation and drinking water services for more than 12 districts of Tamil Nadu. In this study, we have taken the everyday inflow and outflow of the water level at 8.00 am reading in Mettur dam from June 2005 to May 2006. Table 1 presents the average inflow and outflow of water levels (in cusecs) on these months using the data from public work department as exposed below.

Table 1

Year	**Month**	**Average of Water levels (in cusecs)**	
		Inflow	**Outflow**
2005	June	1110	891
	July	12452	2320
	August	34451	15715
	September	20229	23611
	October	39483	34722
	November	23409	22734
	December	9883	10175
2006	January	2890	6364
	February	1071	992
	March	2265	1480
	April	1592	1866
	May	2914	1913

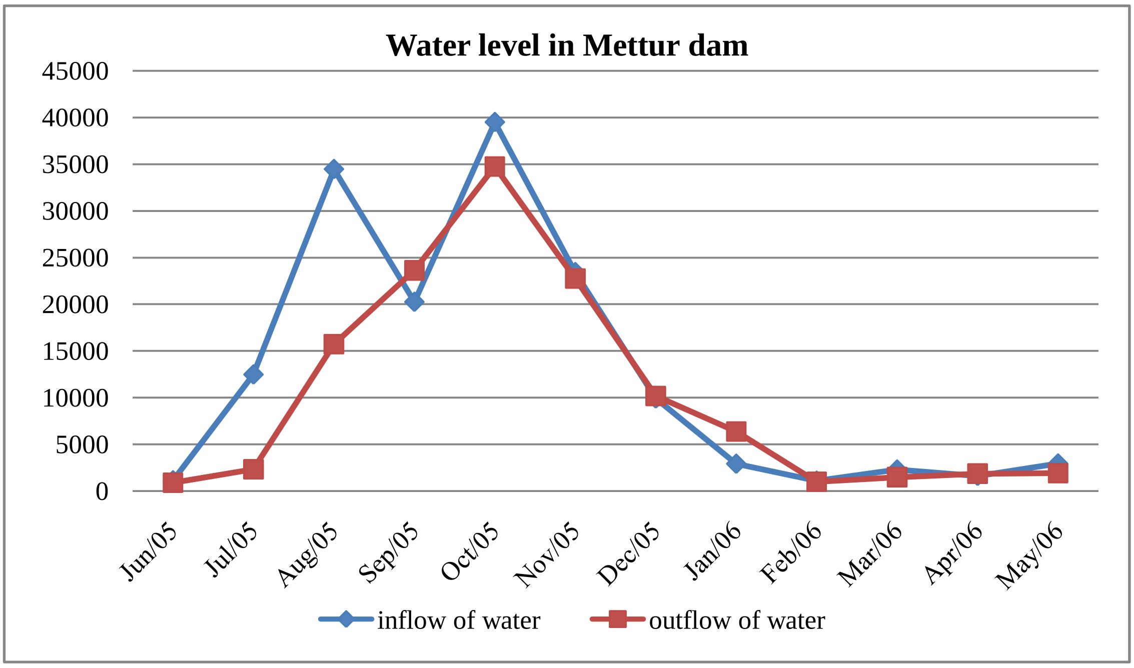

Figure 1

Figure (1) represents the average inflow and outflow of water levels (in cusecs) in Mettur Dam from June 2005 to May 2006.From this Figure (1) we conclude that there was not enough water in the months of June and July. Most of the water has come from August to November. The inflow and outflow of water level reaches its maximum in the month of October. After January 2006 there was no noticeable inflow and outflow of water in Mettur Dam. We apply the points attained from Figure (1) in the SPSS mathematical software and use these values in the properties of Guassian progression we obtain Figure (2).

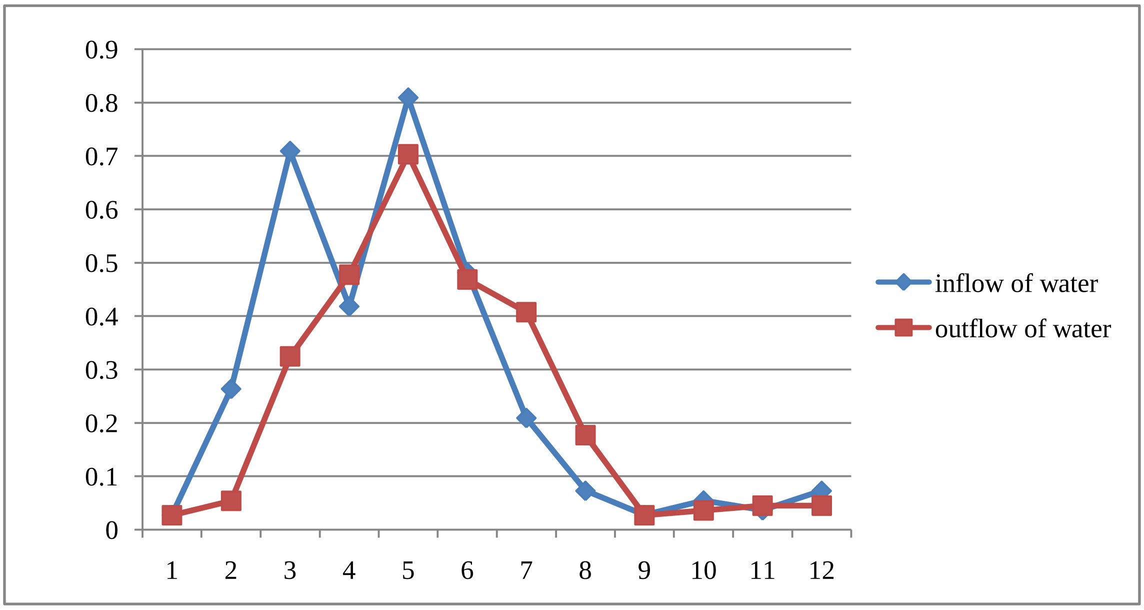

Figure 2

5. Conclusion:

A main portion of water amassed behind dams in the world is reserved for irrigation which regularly comprises consumptive purpose. Since the dams were legally responsible a few decades, for escorting in the green revolution throughout high yielding crops and appliance of fertilizers, instructing food safety in the features of ever mounting population[10]. Mettur dam does not receive much water and nearly goes dry during persuaded stages when the water is most required by the farmers. This study, examined everyday inflow and outflow water in Mettur Dam from June 2005 to May 2006. From figure (1), we infer that, between August 2005 and November 2006, the Mettur Dam received 327 TMC of water approximately. That means an average of 81 TMC of water received every month in this duration. Since Kuruvai cultivation begins in June of every year, it requires more water from June. If this 327 TMC of water had supplied at an average rate of 54 TMC of water per month from June then the Mettur Dam could have been opened in June. This would have caused no loss to the people and the Government. In this paper, the functional convergence theorems for quadratic variations of second order of Gaussian progression with singularity function and the stationary increments of Gaussian progression will be discussed and compared with the water level in Mettur dam. Finally from Figures (1) & (2) we conclude that, the result of the mathematical model identical with the departmental report.

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