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A STOCHASTIC TIME DETERIORATION PROCESS FOR INFLOW OUTFLOW ANALYSIS OF METTUR DAM DURING JUNE 2010 TO MAY 2011

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Abstract:

Agriculture is the backbone of India. Our country's economy mainly depends on agriculture. The main source of agriculture is water .agriculture land gets water through rainfall, Ground water and the outflow of water from dams, in this paper we take out the inflow and outflow of Mettur dam during the year June 2010 to May 2011 and analyzed about the utility of water for agriculture through dam. The stochastic deterioration of gamma process is being applied in this paper for analyzing the inflow and outflow of water in Mettur dam.

Key Words: Water Inflow and Outflow, Reliability, Deterioration & Gamma Process

1. Introduction:

Water is a transparent fluid which forms the world's streams, lakes, oceans and ground water. Human societies are grappling with the need to supply reliable and affordable water to growing populations [1]. Climate change is intensifying this challenge as droughts increase in both frequency and severity in many parts of the world, leading to greater risk of water supply deficits [10, 11]. Innovative strategies are now needed to account for saving water. Water resource management and development are concomitant .Without proper management; the water resource developed can be lost without playing a significant role in crop-production and socio-economic development of the area [8]. In this paper we have given out gamma process with deterioration for analyzing monthly inflow and outflow of water through graph. In gamma process, we have a gamma distribution with shape and scale parameter and also independent non-negative increments.

2. Random-Variable Degradation Model:

The degradation of resistance for reliability is given by

$$R(t) = R_0\left(1 - \frac{A}{R_0 t^{-b}}\right) \qquad (1)$$

WhereR_0 - initial resistance,
A - Random rate of degradation,
t –time interval,
b-a non linear trend of the degradation law

The randomization of the degradation rate reflects its variability in a large population of similar components (in a similar manner as the variability in lifetimes).Generally the failure-limit can be written as $R(t) - S = 0$ .The lifetime distribution can be simply obtained from the relation

$$t = \sqrt[b]{\frac{R_0}{A}\left[1 - \frac{s}{R_0}\right]} \qquad (2)$$

The lifetime distribution is derived depending on the probability distribution of R_0, S and A, For example consider initial resistance r_0 and stress s to be deterministic variables and linear degradation law with the rate A as a gamma distributed random variable. Under these assumptions the lifetime distribution is said to be an inverted gamma distribution. Another example is a lognormal distribution for A as used by Mori & Ellingwood [9].

Although the degradation model in equation (1) can be considered as a stochastic process in technical sense, the sample paths of the degradation of a component remain fixed over its entire lifetime. The single inspection can also determine the remaining lifetime of the component without any uncertainty. If there are n unknowns in the degradation law , n number of inspections will determine the remaining lifetime of a component .For time dependent reliability analysis we apply First–order reliability method (FORM) from random-variable degradation model (RVD).For the optimization of inspection and maintenance programs ,the uncertainty associated with the evolution of degradation over time is an important. In the condition assessment and rehabilitation planning of existing structures, since the RVD model is not adequate to model temporal variability associated with degradation, to overcome this limitation we present a more formal stochastic-process degradation (SPD) model. Here in this the component is regarded as inflow and outflow of water and life time is the whole year.

3. Stochastic-Process Degradation Model:

The process of deterioration is uncertain, so it is difficult in modeling time dependent reliability. The uncertain deterioration is said to be a stochastic process, and the associated uncertainty is represented by the normal distribution. This probability distribution has been used for modeling the exchange value of shares and the movement of small particles in fluids and air. A characteristic feature of this model, also denoted by the Brownian motion with drift (karlin and Taylor)[6], is that a water inflow and outflow alternately increases and decreases ,like the exchange value of share .For this reason ,the Brownian motion is inadequate in modeling deterioration which is monotone . In order to incorporate a monotonic increase in deterioration with time, we propose the gamma process as an ideal alternative (van Noortwijket al.) [13]. Abdel-Hameed [2] was the first to propose the gamma process as a proper model for deterioration occurring random in time. In his paper he called this stochastic process the "gamma wear process". An advantage of modeling deterioration processes through gamma processes is that the required mathematical calculations are relatively straight forward .the mathematical definition of the gamma process is given as follows: a random quantity X has a gamma distribution with shape parameter $v > 0$ and scale parameter $u > 0$ if its probability density function is given by

$$Ga(x/v,u) = \frac{u^{v}}{\Gamma(v)} x^{v\left(1-\frac{1}{v}\right)} e^{\{-ux\}} I_{(0,\infty)}(x)$$

Where $I_A(x) = \begin{cases} 1, \, for x \in A \\ 0, \, for x \notin A \end{cases}$

$\Gamma(a) = \int_{t=0}^{\infty} \frac{t^{a\left(1-\frac{1}{a}\right)}}{e^{t}} dt$ is the gamma function for a>0.

Let $v(t)$ be a non-decreasing, right continuous ,real-valued function for $t \geq 0$ with $v(0) \equiv 0$ the gamma process with shape function $v(t) > 0$ and scalar parameter $u > 0$ is a continuous time stochastic process $\{X(t), t \geq 0\}$.Let X(t) denote the deterioration at time t ,$t \geq 0$,and let the probability density function of X(t), in accordance with the definition of the gamma process be given by

$$f_{x(t)}(x) = Ga(x/v(t),u) \tag{3}$$

With

$$E(X(t)) = v(t)u^{-1} \tag{4}$$

$$Var(X(t)) = v(t)u^{-2} \tag{5}$$

An analysis is said to fail when its deterioration resistance, denoted by $R(t) = r_0 - X(t)$,drops below the stress s. we assume both the initial resistance r_0 and stress s to be deterministic .Let the time at which failure occurs be denoted by the lifetime T. Due to the gamma distributed deterioration , equation (3) ,the lifetime distribution can then be written as:

$$F(t) = \Pr\{T \leq t\} = \Pr\{X(t) \geq r_0 - s\} = \int_{x=r_0\left(1-\frac{s}{r_0}\right)}^{\infty} f_{X(t)}(x)dx = \frac{\Gamma(v(t),[r_0 u - su])}{\Gamma(v(t))} \tag{6}$$

Where $\Gamma(a,x) = \int_{t=x}^{\infty} \frac{t^{a\left(1-\frac{1}{a}\right)}}{e^{t}} dt$ is the incomplete gamma function for $x \geq 0$ and a>0

Using chain rule for differentiation, the pdf of the life time is given by,

$$f(t) = \frac{\partial}{\partial t}\left[\frac{\Gamma(v(t),[r_0 u - su])}{\Gamma(v(t))}\right] = \frac{\partial}{\partial \tilde{v}}\left[\frac{\Gamma(\tilde{v},[r_0 u - su])}{\Gamma(\tilde{v})}\right]\Bigg|_{\tilde{v}=v(t)} v^{'}(t) \tag{7}$$

Under the assumption that the shape function $v(t)$ is differentiable .the partial derivative in equation (7) can be calculated by the algorithm of Moore [7]. Using a series expansion continued fraction expansion, this algorithm computes the first and second partial derivatives with respect to x and a of the incomplete gamma integral

$$P(a,x)=\frac{1}{\Gamma(a)}\int_{t=0}^{x}\frac{t^{a\left(1-\frac{1}{a}\right)}}{e^{t}}dt=\left(1-\frac{\Gamma(a,x)}{\Gamma(a)}\right) \tag{8}$$

Empirical studies show that the expected deterioration at time t is often proportional to a power law:

$$v(t)=\frac{c}{t^{-b}} \tag{9}$$

For some physical constants c>0 and b>0, some examples are given by Ellingwood and Mori (1993) [5]. In the event of expected deterioration in terms of a power law, the parameters c and u yet must be assessed by sing statistics. It should be noted that the gamma process is not restricted to using a power law for modeling the expected deterioration over time. As a matter of fact, any shape function v (t) suffices, as long as it is a non-decreasing, right continuous and real-valued function. The main difference between SPD model and the RVD model is that the sample paths of the former approach are discontinuous and monotone, whereas the sample paths of the latter approach are straight lines. According to the gamma process, one inspection thus reveals only one observed increment which can be used to predict future deterioration .According to the random-variable degradation model, however, one inspection already fixes the future deterioration beforehand. Although the RVD model can be very well used as an approximation, one should be careful as soon as inspections are involved. For inspection models based on the gamma process, see eg van Noortwijk et al [12, 13].

4. Statistical Estimation:

In order to apply the gamma process model to practical examples ,statistical methods for the parameter estimation of a gamma process are required .A typical data set consists of inspection times $t_i, i=1,\ldots,n$ where $0=t_0<t_1<t_2<\ldots<t_n$ and corresponding observations of the cumulative amounts of deterioration $x_i, i=1,\ldots,n,$ where $0=x_0\le x_1\le\ldots\le x_n$. consider a gamma process with shape function $v(t)=\frac{c}{t^{-b}}$ and scale parameter u .we assume that the value of the power b is known ,but c and u are unknown. the two most common methods of parameter estimation are (i)Maximum likelihood (ii)Method of moments. Both methods or deriving the estimators of c and u were initially presented by cinlar t al [4].

4.1 Method of Maximum Likelihood:

The maximum-likelihood estimators of c and u can be obtained by maximizing the logarithm of likelihood function of the increments .The likelihood function of the observed deterioration increments $\delta_i=x_i-x_{i-1}, i=1,\ldots,n$ is a product of independent gamma densities

$$l(\delta_1,\ldots,\delta_n/c,u)=\prod_{i=1}^{n}f_{X(t_i)-X(t_{i-1})}(\delta_i)=\prod_{i=1}^{n}\frac{u^{c[t_i^b-t_{i-1}^b]}}{\Gamma(c[t_i^b-t_{i-1}^b])}\delta_i^{[t_i^b-t_{i-1}^b]-1}e^{[-u\delta_i]} \tag{10}$$

it follows that the maximum likelihood estimator of u is $\hat{u}=\hat{c}t_n^b(x_n)^{-1}$ where $\hat{c}$ must be computed iteratively. Given the maximum –likelihood estimator of u, the expected deterioration at time t can be written as

$$E(X(t))=x_n\left[\frac{t_n}{t}\right]^{-b}$$

Because cumulative amounts of deterioration are measured, the last inspection contains the most information. The expected deterioration at the last inspection (at time t_n) equals x_n; (i.e.) $E(X(t_n))=x_n$ .By taking, $\hat{u}=\hat{c}t_n^b(x_n)^{-1}$ the maximum likelihood estimate of c can be obtained by solving the following equation for c:

$$\frac{\partial}{\partial c}\log(\delta_1,\ldots,\delta_n|c)=\sum_{i=1}^{n}\frac{\partial}{\partial c}\left\{c[t_i^b-t_{i-1}^b][\log(ct_n^b)-\log[x_n]]-\log\Gamma(c[t_i^b-t_{i-1}^b])++(c[t_i^b-t_{i-1}^b]-1)\log\delta_i-ct_n^b(x_n)^{-1}\delta_i\right\}$$

$$=\sum_{i=1}^{n}\left\{[t_i^b-t_{i-1}^b]\left[(\log ct_n^b-\log x_n)+1-\psi(c[t_i^b-t_{i-1}^b])+\log\delta_i\right]-t_n^b(x_n)^{-1}\delta_i\right\}$$

$$=t_n^b\left(\log ct_n^b-\log x_n\right)+\sum_{i=1}^{n}\left[t_i^b-t_{i-1}^b\right]\left\{\log\delta_i-\psi\left(c\left[t_i^b-t_{i-1}^b\right]\right)\right\}=0 \qquad (11)$$

Where the function $\psi(a)$ is the derivative of the logarithm of the gamma function:

$$\psi(a)=\frac{\Gamma'(a)}{\Gamma(a)}=\frac{\partial\log\Gamma(a)}{\partial a} \qquad a>0$$

$\psi(a)$ is called the digamma function and can be accurately computed using the algorithm developed by Bernado [3] .Summarizing ,the maximum-likelihood estimates $\hat{c}$ and $\hat{u}$ can be solved from

$$\hat{u}=\hat{c}t_n^b\left(x_n\right)^{-1},\ \sum_{i=1}^{n}\left[t_i^b-t_{i-1}^b\right]\left\{\psi\left(\hat{c}\left[t_i^b-t_{i-1}^b\right]-\log\delta_i\right)\right\}=t_n^b\left[\log\hat{c}t_n^b-\log x_n\right] \qquad (12)$$

4.2 Methods of Moments:

The expected value and variance of the accumulated deterioration at calendar time t are given by

$$E(X(t))=\frac{cu^{-1}}{t^{-b}}\ Var(X(t))=\frac{cu^{-2}}{t^{-b}} \qquad (13)$$

when the power b is known ,for transformation of non-stationary gamma process to a stationary gamma process we use a monotonic transformation from the clock or calendar time t to the transformed or operational time $z(t)=t^b$. A stochastic process has stationary increments if the probability distribution of the increments $X(t+h)-X(t)$ depends only on h for all $t,h\geq 0$. Substituting the inverse time transformations of $z(t)=t^b,(i.e.)\ t(z)=z^{b^{-1}}$ in eq 13 yields

$$E(X(t(z)))=czu^{-1}\ Var(X(t))=\frac{cu^{-2}}{t^{-b}} \qquad (14)$$

equation (14) results in a stationary gamma process with respect to the transformed time .similarly, the transformed inspection times $z_i=t_n^b,i=1,\ldots..,n.$ Let us further define the transformed times between inspections as $w_i=t_i^b-t_{i-1}^b,i=1,\ldots.,n$ and for mathematical convenience

$$D_i=X_i-X_{i-1}\quad,\ Y_i=D_i-cw_iu^{-1}\quad,\ i=1,\ldots,n \qquad (15)$$

The deterioration increment D_i has a gamma distribution with shape parameter cw_i and scale parameter u for all i=1, . . . ,n ,and the increments D_1 , ...,D_n are independent .Note that X_i,Y_i
And D_i denote random quantities and x_i,δ_i and y_i the corresponding observations .For each i ,the first and second moment of Y_i are

$$E(Y_i)=0\ ,\quad E(Y_i^2)=cw_iu^{-2} \qquad (16)$$

We introduce the following average rates of deterioration per unit of transformed time:

$$\bar{D}=\sum_{i=1}^{n}D_i\left(\sum_{i=1}^{n}w_i\right)^{-1}\quad,\quad\bar{Y}=\sum_{i=1}^{n}Y_i\left(\sum_{i=1}^{n}w_i\right)^{-1}\quad,\quad\bar{Y}=\bar{D}-cu^{-1} \qquad (17)$$

From equation (16) and (17), it follows that

$$E(\bar{Y})=0\,,E(\bar{Y}^2)=\sum_{i=1}^{n}E(y_i^2)\left(\sum_{i=1}^{n}w_i\right)^{-2}=cu^{-2}\sum_{i=1}^{n}w_i\left(\sum_{i=1}^{n}w_i\right)^{-2}=cu^{-2}\left(\sum_{i=1}^{n}w_i\right)^{-1} \qquad (18)$$

Now, one may calculate $E(\bar{D})=cu^{-1}$ and equation (15) yields

$$\sum_{i=1}^{n}\left(D_i-\bar{D}_{w_i}\right)^2=\sum_{i=1}^{n}\left[D_i-cu^{-1}w_i-\left(\bar{D}-cu^{-1}\right)w_i\right]^2=\sum_{i=1}^{n}\left[Y_i-\bar{Y}w_i\right]^2 \qquad (19)$$

Because $E(Y_i)=0$, the second term in the last sum can be rewritten as

$$E(Y_i\bar{Y})=E\left(Y_i\sum_{j=1}^{n}Y_j\right)\left(\sum_{i=1}^{n}w_i\right)^{-1}=E\left(Y_i^2+Y_i\sum_{i\neq j}Y_i\right)\left(\sum_{i=1}^{n}w_i\right)^{-1}=E(Y_i^2)\left(\sum_{i=1}^{n}w_i\right)^{-1}=cu^{-2}w_i\left(\sum_{i=1}^{n}w_i\right)^{-1} \qquad (20)$$

By taking expectations on both sides of equation (19) and by applying eq(18-20) ,it follows that

$$E\left(\sum_{i=1}^{n}\left(D_i - \bar{D}w_i\right)^2\right) = cu^{-2}\left(\sum_{i=1}^{n} w_i - \sum_{i=1}^{n} w_i^2\left(\sum_{i=1}^{n} w_i\right)^{-1}\right) \tag{21}$$

Finally, the method of moments estimates $\hat{c}$ and $\hat{u}$ can be solved from

$$\hat{c}\hat{u}^{-1} = \sum_{i=1}^{n}\delta_i\left(\sum_{i=1}^{n} w_i\right)^{-1} = x_n\left(t_n^b\right)^{-1} = \bar{\delta}\ ,\ \hat{c}\hat{u}^{-2}\left(\sum_{i=1}^{n} w_i - \sum_{i=1}^{n} w_i^2\left(\sum_{i=1}^{n} w_i\right)^{-1}\right) = \frac{1}{\sum_{i=1}^{n}\left(\delta_i - \bar{\delta}w_i\right)^{-2}} \tag{22}$$

Or, equivalently

$$\hat{c}\hat{u}^{-1} = \sum_{i=1}^{n}\delta_i\left(\sum_{i=1}^{n} w_i\right)^{-1} = x_n\left(t_n^b\right)^{-1} = \bar{\delta}\ ,\ x_n\hat{u}^{-1}\left(1 - \sum_{i=1}^{n} w_i^2\left(\sum_{i=1}^{n} w_i\right)^{-2}\right) = \frac{1}{\sum_{i=1}^{n}\left(\delta_i - \bar{\delta}w_i\right)^{-2}} \tag{23}$$

Here, the method of moments leads to simple formulae for parameter estimation. The first equation in the maximum-likelihood estimation (12) is same as the first equation in the method of moment estimation (23)

5. Simulation Study:

We have collected the data of inflow and outflow of water in the Mettur dam during the period June 2010 to May 2011 and examined. On calculating the mean of inflow and outflow of water for every fifteen days we get 24 sets of data, then we plot the graph for inflow and outflow of water that is given in fig (1). We analyze the graph in spss software for fitting data to mathematical model .Then we use time deterioration model in random variable and stochastic process and also given two methods in time deterioration for estimating the parameters and the results determined in both methods are same. Then we plot the graph mathematically and it is given in fig (2).we then examines the inflow and outflow of water in both the graphs.

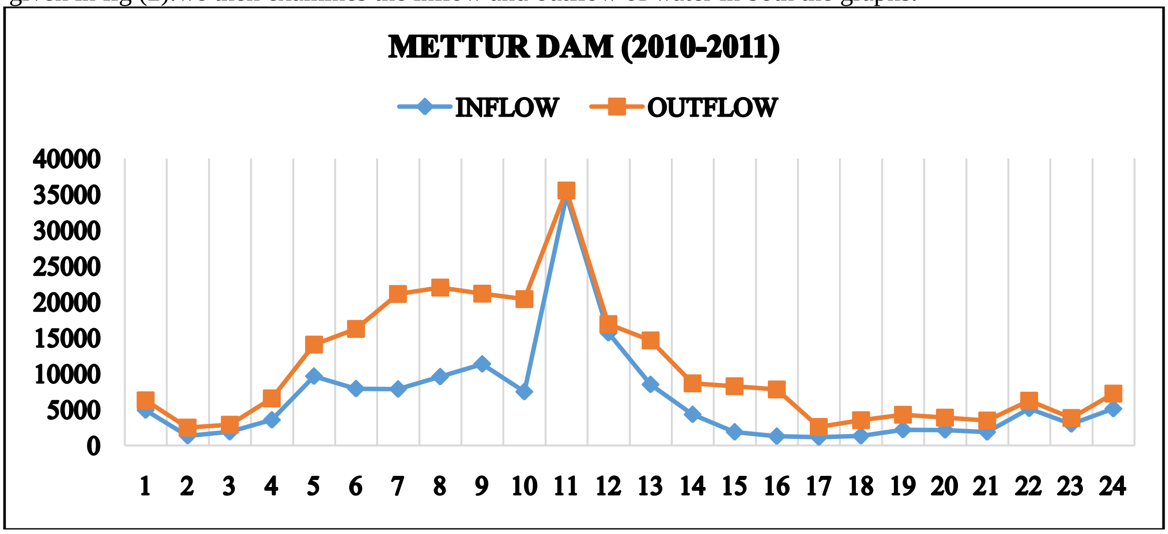

Figure 1: A Line graph representing inflow and outflow of water using original data

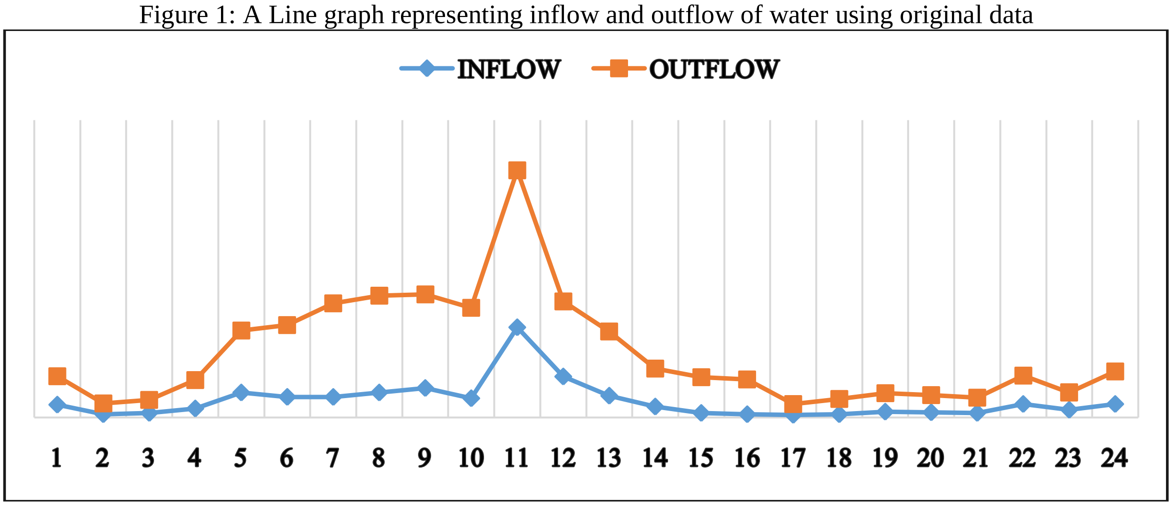

Figure 2: A line graph representing inflow and outflow of water using time deterioration

6. Conclusion:

In Tamilnadu agriculture has a great loss now days due to the unavailability of water, Other than the farmers who have bore wells and drip irrigation facilities, the poor farmers totally depend on the water from Mettur dam and rainfall. During June 2010 to May 2011 annually near 214 TMC of water had been received by Mettur dam and approximately 152 TMC of water had been released from Mettur dam. Between August 2010 and December 2010 approximately 110 TMC of water was released, during these months nearly 24 TMC of water had been dropped into sea without any use. If this 24 TMC of water had been released during the month of June, the farmers would have started their cultivation earlier which in turn increases food production and economy of Tamilnadu. In this paper we have used time deterioration of gamma process along with their various methods of parameter estimation and analyzed the inflow and outflow of water mathematically. Finally we conclude that departmental data (fig 1) agree with the mathematical data (fig2)

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